

Correcting $\alpha_3(M_Z)$ in the NMSGUT

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Abstract

We show that superheavy threshold corrections in the New Minimal Supersymmetric GUT based on the SO(10) Higgs system $\mathbf{210} \oplus \mathbf{126} \oplus \overline{\mathbf{126}} \oplus \mathbf{10} \oplus \mathbf{120}$ can comfortably correct the prediction for the value of $\alpha_3(M_Z)$ from the relatively large value predicted by the two loop RG equations to the central value determined by the current world average. The unification scale is raised above the one loop value over almost all of the viable parameter space.

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1 Introduction

Since the discovery of neutrino oscillations renormalizable Supersymmetric SO(10) GUTs[1, 2, 3, 4, 5, 6, 7] have increasingly come to occupy the middle ground of the Grand Unification landscape due to their natural accommodation of seesaw mechanisms for neutrino masses and their ability to justify R-parity as a part of the gauge symmetry[8] while preserving it unbroken to low energies[9, 3] when the most economical and viable Higgs set i.e $\mathbf{126} \oplus \overline{\mathbf{126}} \oplus \mathbf{210}$ is used. They thus also predict a stable LSP which is welcome as cosmological dark matter. The ability of these theories to accommodate all the known (and probably also all the as yet unmeasured) fermion mass-mixing data has also been demonstrated *provided* the full complement of Fermion mass (FM) Higgs allowed by group theory i.e $\mathbf{10}, \mathbf{120}, \overline{\mathbf{126}}$ are used[11, 13, 14, 15, 16], because the model without the $\mathbf{120}$ *fails*[10, 11, 12] : thus providing a welcome indication that it has matured sufficiently as a scientific hypothesis so as to be vulnerable to falsification.

Given these successes it is natural to ask whether -and with what additional constraints on its parameters- this so called New Minimal Susy GUT (NMSGUT)[7] is able to account for the longstanding discrepancy between the increasingly accurate experimental estimates of the SM gauge couplings at M_Z and the values predicted by Renormalization Group flows in the NMSGUT. We have shown that the effects of threshold corrections due to superheavy particles are modest and compatible with the one loop architecture of MSSM coupling unification[6, 7]. Moreover we found[7] that the bulk of the viable parameter space(according to our 10% error allowances) implied that the unification scale(and with it all other superheavy masses) was raised by about one order of magnitude or more thus alleviating the crisis regarding the too short $d = 5$ proton decay lifetime estimates in Susy GUTs.

However our analysis of the RG constraints followed a somewhat non standard format. We calculated the effects of threshold corrections on the values of $\sin^2 \theta_W(M_S), \alpha_G, M_{GUT}$ rather than the standard choices $\sin^2 \theta_W(M_Z), \alpha_G, M_{GUT}$ (given $\alpha_3(M_Z)$) or $\alpha_3(M_Z), \alpha_G, M_{GUT}$ (given $\sin^2 \theta_W(M_Z)$). We concluded that allowing for uncertainties $\sim 10\%$ in our knowledge of the unification parameters restricted, but by no means eliminated, the parameter space of this class of Supersymmetric GUTs. However the difference from the accepted format has led to some difficulty in communicating our results to workers in the field. Therefore in this letter we carry out a precision analysis in terms of the superheavy threshold corrections to the prediction of $\alpha_3(M_Z), \alpha_G, M_{GUT}$.

Our analysis is aimed at restricting the parameter space of the NMSGUT based on the requirement that the parameters be such as to yield a lowering of the prediction of $\alpha_3(M_Z)$ from the two loop corrected value of 0.130 which is uncomfortably larger than the central value of the world average experimental value $0.120 \pm .01$. We indicate the region of the control parameter space of the NMSGUT (as well as its complex parameter cousin) which yields threshold corrections of the right sign and magnitude to bring the prediction for $\alpha_3(M_Z)$ back to the observed central value.

2 Threshold corrections

The two loop RG flow equations for the three gauge couplings of the standard model and MSSM can be integrated[17, 18, 19] to yield predictions for the Grand Unification scale, the value of the gauge coupling at unification and the value of one of the gauge couplings at the scale M_Z given the values of the other two as inputs. Since it is $\alpha_3(M_Z)$ which carries the largest uncertainty ($\sim 8\%$) while $\alpha_{em}(M_Z), \sin^2 \theta_W(M_Z)$ are quite precisely known (better than 0.01%, 0.1% respectively) it is usual[18, 19] to choose to predict $\alpha_3(M_Z)$. Using updated parameter values[20]

$$\begin{aligned} M_H &= 117 \text{ GeV} & ; & & M_Z &= 91.1876 \pm .0021 \text{ GeV} \\ \alpha(M_Z)^{-1} &= 127.918 \pm .018 & ; & & \hat{s}_Z^2 &= .23122 \pm .00015 \\ m_{pole}^t &= 172.7 \pm 2.9 \text{ GeV} \end{aligned} \quad (1)$$

we find from the equations of [19]

$$\alpha_s(M_Z) - \Delta_{\alpha_s} = 0.130 \pm 0.001 + 3.1 \times 10^{-7} \text{ GeV}^{-2} \times [(m_t^{pole})^2 - (172.7 \text{ GeV})^2] + H_{\alpha_s} \quad (2)$$

where $\Delta_{\alpha_s} = \Delta_{\alpha_s}^{GUT} + \Delta_{\alpha_s}^{Susy}$ threshold corrections.

The effect of the two loop Yukawa coupling corrections H_{α_s} was estimated[19] to be bounded $:-0.003 < H_{\alpha_s}(h_t, h_b) < 0$

Thus if we place credence on the central value of 0.12 for $\alpha_s(M_Z)$, the effects of the low energy thresholds (equivalently parameterized by an effective supersymmetry breaking scale M_{SUSY} [19]), GUT thresholds (and even NRO corrections) lumped together into Δ_{α_s} should correct the excessive value found. The effect of the Susy thresholds can raise or lower the value of $\alpha_s(M_Z)$. For $250 \text{ GeV} > M_{SUSY} > 20 \text{ GeV}$ (the actual values of superpartner masses will be much higher since M_{SUSY} is a composite parameter formed from all the superpartner masses[19]) one finds that $0.005 > \Delta_{\alpha_s}^{Susy} > -0.003$. Thus it appears that $\alpha_s(M_Z) - \Delta_{\alpha_s}^{GUT}$ could be as high as 0.135 or as low as 0.124 so that superheavy threshold corrections in the range $-0.004 > \Delta_{\alpha_s}^{GUT} > -0.015$ are indicated.

As we have described at length in earlier papers[6, 10, 11, 7], the superheavy thresholds are controlled by one ‘fast’ control parameter x and several other ‘slow’ parameters which are generically of order 1 or else do not affect the thresholds significantly. Thus it is possible to scan over the complex x plane calculating the changes in the RG flow predictions due to the GUT thresholds and obtain an overview of the behaviour over the whole parameter space. By imposing stability of the one loop analysis the viable ranges of x and even of the slow parameters can be depicted[6, 11, 7]. In earlier papers we had taken $\alpha_s(M_S)$ as input and restricted changes in the output $\sin^2 \theta_W(M_S)$ to be no more than 10%. We then obtained a relatively wide range of viable parameter values. With $\sin^2 \theta_W(M_Z)$ as input and the narrow range of desired threshold corrections as described above one gets a relatively narrow range of permitted values of the fast parameter x . However it should be remembered that the x parameter is determined by solving a cubic equation with one parameter ξ which is a ratio $\xi = \lambda m / \eta M$ of the MSGUT parameters, so that even a fixed value of x would allow a ball of the actual parameter values. In[7] we had advocated a version of the theory with real parameters only (with CP violation arising spontaneously).

In that case one needs to scan only over a parameter line rather than the complex plane. In the next section we briefly recapitulate the structure of NMSGUT to make the RG analysis intelligible.

The mass formulae required to compute the threshold corrections are all given in [6, 7]. We do not repeat them here but simply furnish plots illustrating the restriction of the parameter space due to the above described demands placed on it .

3 Essentials of NMSGUT

The NMSGUT [7] is a renormalizable globally supersymmetric $SO(10)$ GUT whose Higgs chiral supermultiplets consist of AM(Adjoint Multiplet) type totally antisymmetric tensors : $\mathbf{210}(\Phi_{ijkl})$, $\overline{\mathbf{126}}(\overline{\Sigma}_{ijklm})$, $\mathbf{126}(\Sigma_{ijklm})(i, j = 1 \dots 10)$ which break the GUT symmetry to the MSSM, together with Fermion mass (FM) Higgs $\mathbf{10}(\mathbf{H}_i)$ and $\mathbf{120}(O_{ijk})$. The $\overline{\mathbf{126}}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three $\mathbf{16}$ -plets $\Psi_A (A = 1, 2, 3)$ contain the matter including the three conjugate neutrinos ($\bar{\nu}_L^A$). The superpotential (see [4, 21, 5, 6, 7] for comprehensive details) contains the mass parameters

$$m : \mathbf{210}^2; \quad M : \mathbf{126} \cdot \overline{\mathbf{126}}; \quad M_H : \mathbf{10}^2; \quad m_O : \mathbf{120}^2 \quad (3)$$

and trilinear couplings

$$\begin{aligned} \lambda &: \mathbf{210}^3; & \eta &: \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}}; & \gamma \oplus \bar{\gamma} &: \mathbf{10} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ k &: \mathbf{10} \cdot \mathbf{120} \cdot \mathbf{210}; & \rho &: \mathbf{120} \cdot \mathbf{120} \cdot \mathbf{210} \\ \zeta &: \mathbf{120} \cdot \mathbf{210} \cdot \mathbf{126}; & \bar{\zeta} &: \mathbf{120} \cdot \mathbf{210} \cdot \overline{\mathbf{126}} \end{aligned} \quad (4)$$

In addition one has two symmetric matrices h_{AB}, f_{AB} of Yukawa couplings of the the $\mathbf{10}, \overline{\mathbf{126}}$ Higgs multiplets to the $\mathbf{16.16}$ matter bilinears and one antisymmetric matrix g_{AB} for the coupling of the $\mathbf{120}$ to two $\mathbf{16}$ s. It was shown [14, 15] that with only spontaneous CP violation, i.e with all the superpotential parameters real, it is till possible to achieve an accurate fit of all the fermion mass data which furthermore evades the difficulties encountered in accommodation with the high scale structure of the MSGUT [11] provided [13, 16, 7, 14, 15, 22] one takes the $\mathbf{10}, \mathbf{120}$ yukawa couplings to be much larger than those of the $\overline{\mathbf{126}}$ so that Type I neutrino masses are enhanced.

The GUT scale vevs and therefore the mass spectrum are all expressible in terms of a single complex parameter x which is a solution of the cubic equation

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2 \quad (5)$$

where $\xi = \frac{\lambda M}{\eta m}$.

Spontaneous CP violation implies that x must lie [7] on one of the two complex solution branches $x_{\pm}(\xi)$, ($\xi \in (-27.917, \infty)$). Since λ, η are already counted as independent $x_{\pm}(\xi)$ counts for M/m .

The other parameters besides x (or equivalently ξ) have a much weaker effect on the spectra and therefore the threshold corrections and can thus be fixed at representative

values ~ 1 while scanning the behavior over the x-plane. Once the viable regions are identified the limits of permissible variation in these slow parameters can be explored[7].

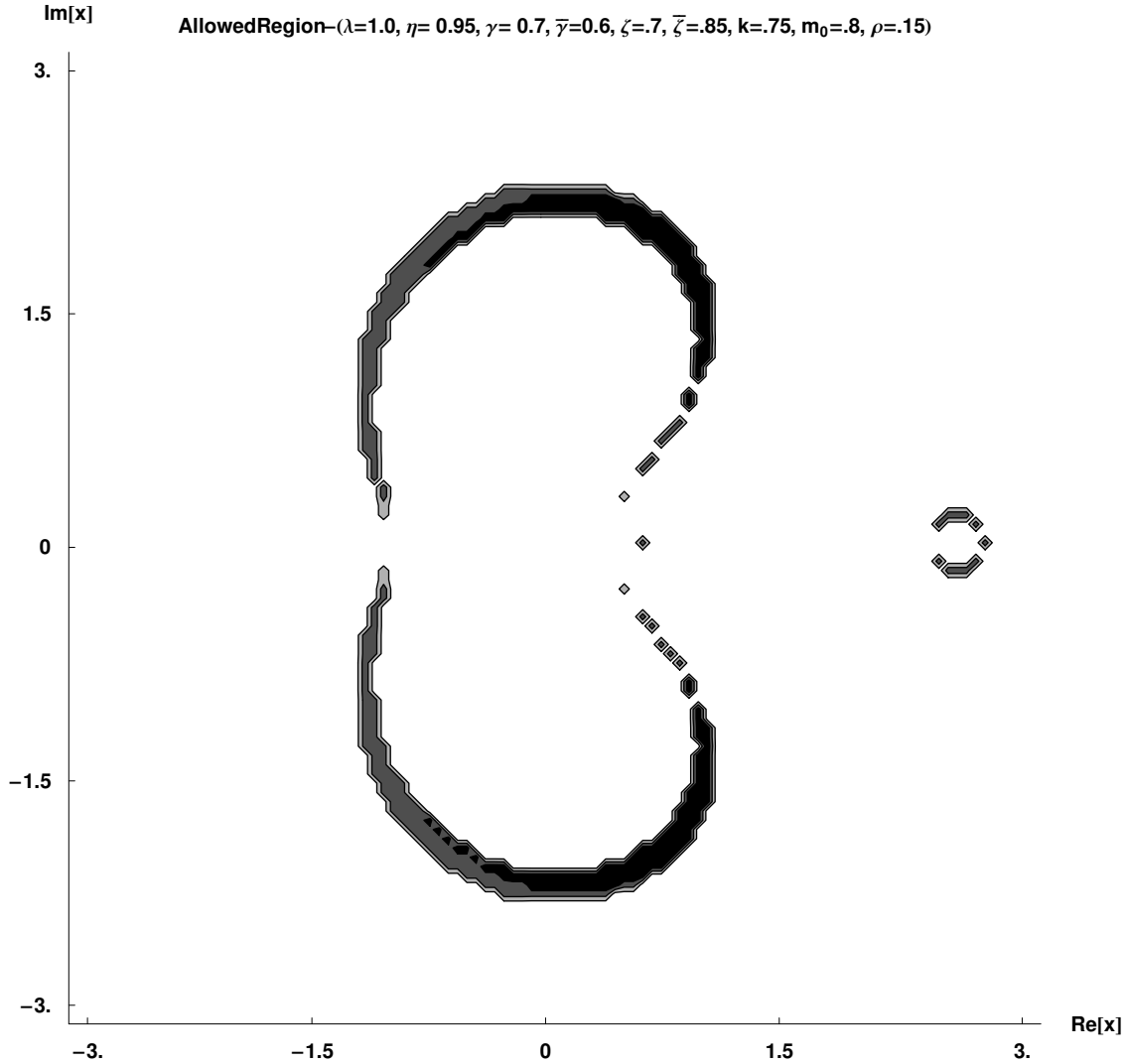


Figure 1: The allowed region(other than white) for threshold corrections in the range $-0.015 < \alpha_3(M_Z) < -0.005$ and for which the changes in α_G, M_X are acceptable. The black region corresponds to $1 \leq \Delta_X \leq 2$, and dark grey to $0 \leq \Delta_X < 1$, while the thin light grey rim is for $-1 \leq \Delta_X < 0$.

4 Correcting $\alpha_3(M_Z)$

In our previous work we had taken it as self evident that, given our demonstration that corrections to the unification parameters could be quite modest ($\sim 10\%$), there would be no problem in accommodating any particular desired values of corrections in this range,

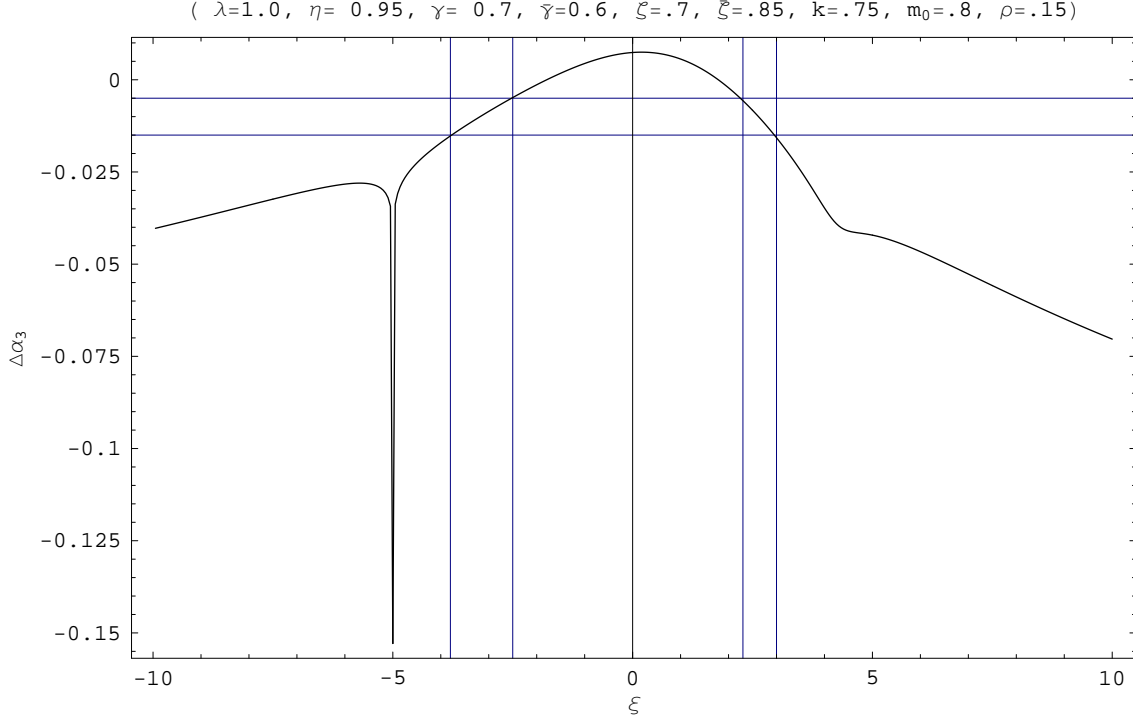


Figure 2: Plot of $\Delta\alpha_3(M_Z)$ against ξ on the CP violating solution branch $x_+(\xi)$ at representative values of the diagonal parameters. Horizontal lines mark out where $\Delta\alpha_3(M_Z)$ is in the desired range $-0.015 < \Delta\alpha_3(M_Z) < -0.005$ and determine the range of viable ξ ($\xi \in (-3.8, -2.5) \cup (2.4, 3)$).

given the number of parameters that affected the superheavy mass spectrum. In other words fixing the GUT superpotential parameters for the AM Higgs fields on the basis of unification requirements alone seems a vain hope since there is likely to remain a significant multi parameter ball compatible with current measured values and limits. However we have recently shown[23] that in fact proper numerical fits of the fermion data actually determine complete candidate NMSGUT parameter sets. Smoking gun discoveries of supersymmetry and GUT characteristic processes such as proton decay will, however, still be necessary to choose among candidate fits and thus actually fix the NMSGUT parameters. Here we merely restrict ourselves to displaying examples of parameter regions where the threshold

corrections are capable of lowering $\alpha_3(M_Z)$ to the current central experimental values while maintaining the very accurately known $\alpha_{em}(M_Z), \sin^2 \theta_W(M_Z)$ at the known central values and limiting the variation in α_G to 10% and $2 > \Delta \text{Log}_{10} M_X > -1$. *Note that although a negative change of 10% in $\text{Log}_{10} M_X$ would be disastrous such a problem in fact never arises : in all cases the region allowed by the other parameters always features a raised value of M_X .* As we have already emphasized previously[7, 22] this raise is in fact a welcome mollification of the mounting tension regarding too large $d = 5$ operator mediated proton decay rates. All RG formulae and mass spectra required for our analysis have already appeared[21, 6, 7] and will not be repeated here.

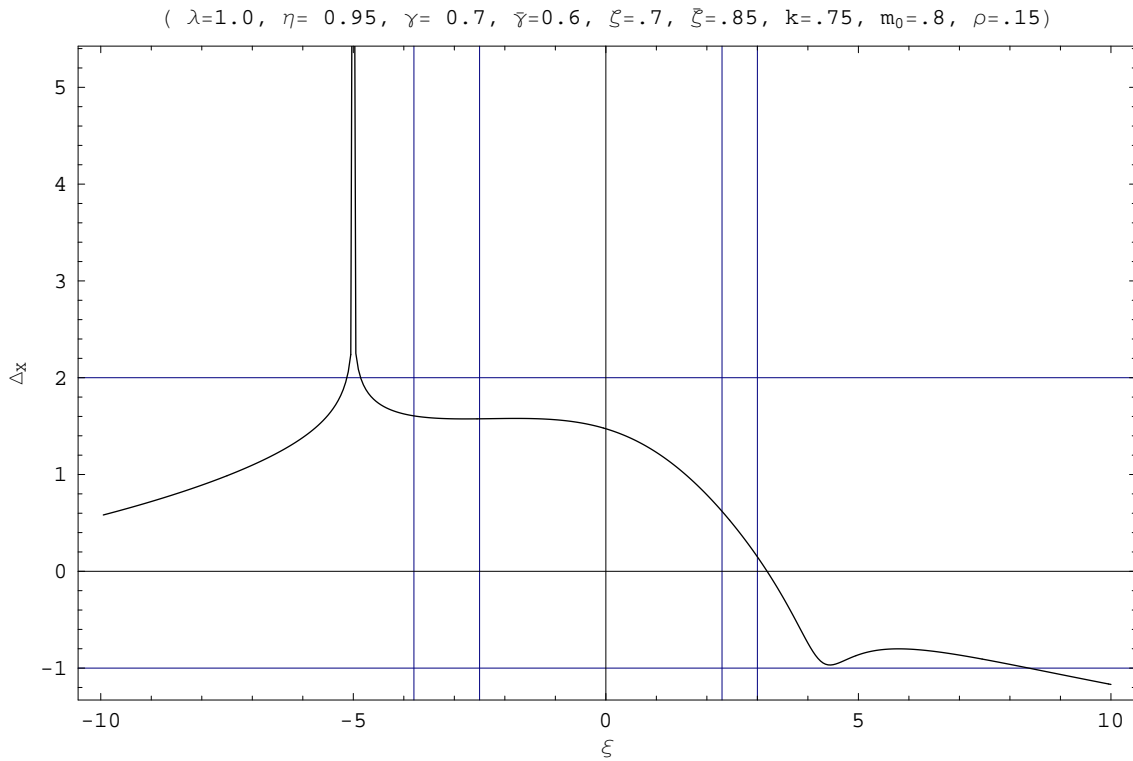


Figure 3: Plot of $\Delta_X = \Delta \log_{10} M_X$ against ξ on the CP violating solution branch $x_+(\xi)$ at representative values of the diagonal parameters. The regions with desired $\Delta\alpha_s(M_Z)$: $-0.015 < \alpha_3(M_Z) < -.005$ are marked with vertical pairs of lines and have M_X raised above the one loop value.

In Figure 1 we plot the region of the x -plane which can give threshold corrections to $\alpha_3(M_Z)$ of the desired size while maintaining acceptable values for α_G, M_X . As already noted in [7] the unification scale is significantly raised over almost the entire allowed range. This can be seen in Figure 1 where the region with $-0.015 < \Delta\alpha_3(M_Z) < -0.005$ is stratified according to the predicted values of M_X .

In [7] we had identified the contour $x_+(\xi)|\xi \in [-27.917, \infty)$ in the x plane corresponding to a complex solution of the cubic equation for x (with positive imaginary part) as the appropriate locus on which to examine the behaviour of a theory in which CP violation arose spontaneously even though initial values of all parameters were real. In practice a plot for $\xi \in (-10, 10)$ is sufficient [7]. In the present context we wish to indicate which parts of the contour allow acceptable threshold corrections for $\alpha_3(M_Z)$ and the values of α_G, M_X are within 10% of their one loop values. In Figure 2 we plot the superheavy threshold correction to $\alpha_3(M_Z)$ versus the parameter ξ on the branch $x_+(\xi)$. In Figure 3 we plot $\Delta \log_{10} M_X$ versus ξ .

From these figures it is apparent that there are significant regions of parameter space where the threshold corrections are of the right magnitude and sign to correct the calculated 2-loop value of $\alpha_3(M_Z)$ to its observed value. In particular we find that the value of M_X is always raised above the one loop unification scale. Moreover this change is accompanied [7] by an parallel raising of all superheavy masses so that the dimension 5 operator mediated proton decay rates are reduced.

5 Discussion

In this letter we have illustrated how not only the general NMSGUT based on the Higgs system i.e. $\mathbf{126} \oplus \overline{\mathbf{126}} \oplus \mathbf{210} \oplus \mathbf{120} \oplus \mathbf{10}$, but also its more constrained but fully realistic version with only real parameters, are comfortably able to provide the threshold corrections of the right magnitude to remove the discrepancy between RG corrected and experimental $\alpha_3(M_Z)$. In fact these constraints from the RG analysis must be confronted [23, 24] with the determination of GUT parameters that arises whenever a fit of the fermion data extrapolated to M_X is performed. The question of whether otherwise viable fits which will determine all parameters and thus threshold corrections will also keep $\Delta\alpha_3(M_Z)$ in desired range is thus amenable to resolution in coming years.

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